International Journal of Advanced Research in Engineering Technology & Science

ISSN: 2349-2819

Email: editor@ijarets.org Volume-8, Issue-9 September – 2021 www.ijarets.org

An Exploration on the Bitopological studies

Sanskriti

M.Phil, Roll No: 141429

Session: 2014-15

University Department of Mathematics

B.R.A Bihar University, Muzzaffarpur

Abstract

In this paper, we have introduced the idea on neutrosophic bitopological space and studied its properties with models. All we have defined a few definitions of neutrosophic interior, closure and limit likewise we have studied its properties.

Keywords: Neutrosophic Closed set; Neutrosophic Open set, Bitopological study

Introduction

In 1995 neutrosophic set has been proposed by F. Smarandache as another part of philosophy dealing with ancient roots, origin, nature and extent of neutralities as well as their interactions with different ideational spectra. The expression "neutron-sophy" signifies information on unbiased contemplations with regular represents the main distinction between fluffy set and intuitionistic fluffy set. In 1965, L. A. Zadeh defined the idea of membership function and discovered the fluffy set. Zadeh explained the idea of uncertainty. In 1989, K. T. Atanassov generalized the ideas of fluffy set and introduced the level of non-membership as an independent part and proposed the intuitionistic fluffy set. After the introduction of fluffy sets, a few explores were directed on the generalizations of the notions of fluffy set. After the generalization of fluffy sets, numerous analysts have applied generalization of fluffy set hypothesis in many parts of science and innovation. Chang introduced fluffy topology. Coker (1997) defined the notion of intuitionistic fluffy topological spaces. In 1963, J.C. Kely defined the investigation of Bitopological spaces. A. Kandil et al discussed on fluffy bitopological spaces. Lee et al. discussed on certain properties of Intuitionistic Fluffy Bitopological Spaces. Presently a day numerous scientists have studied topology on neutrosophic sets, for example, Lupianez and Salama. Abdel-Baset et al. discussed on Hybride plitogenic decision-making approach with quality function sending for selecting inventory network sustainability metrics. As of late Abdel-Baset et al. studied on Novel plithogenic TOPSIS-CRITIC model for sustainable store network risk the executives. In this paper, we introduce the idea of Netrosophic Bitopological Spaces. Then, we introduce the ideas of neutrosophic interior set, neutrosophic closure put down and neutrosophic limit set. Additionally, we have discussed a few propositions connected with neutrosophic interior set, neutrosophic closure put down and neutrosophic limit set.

Basic Concepts

Definition 2.1 [20] A neutrosophic set A on the universe of discourse X is defined as

$$A = \{< x, \mu_{A,}\ \sigma_A,\ \gamma_A >: x \epsilon X\}$$

Where $\mu_{A,}\ \sigma_A,\ \gamma_A : X \longrightarrow]0^-, 1^+[$ and $0^- \leq \mu_A + \sigma_A + \gamma_A \leq 3^+$, μ_A represents degrees of membership function, σ_A is the degree of indeterminacy and γ_A is the degree of non-membership function.

Let $= \{< x, \mu_{A,}\ \sigma_A,\ \gamma_A >: x\epsilon X\}$ and $B = \{< x, \mu_{B,}\ \sigma_B,\ \gamma_B >: x\epsilon X\}$ be two neutrosophic sets on X. Then

i. Neutrosophic subset: A≤B if $\mu_A \leq \mu_B$, $\sigma_A \geq \sigma_B$ and $\gamma_A \geq \gamma_B$, That is A is neutrosophic subset of B

ii. Neutrosophic equality: If A ≤B and A ≥B then A=B

iii. Neutrosophic intersection : A∧B = $\{< x, \mu_A \wedge \mu_A,\ \sigma_A \vee \sigma_A,\ \gamma_A \vee \gamma_B >: x\epsilon X\}$

iv. Neutrosophic union: A∨B = $\{< x, \mu_A \vee \mu_A,\ \sigma_A \wedge \sigma_A,\ \gamma_A \wedge \gamma_B >: x\epsilon X\}$

v. Neutrosophic complement: $A^C = \{< x, \gamma_A,\ 1 - \sigma_A,\ \mu_{A,} >: x\epsilon X\}$

vi. Neutrosophic universal set: $1_X = \{< x,\ 1,\ 0,\ 0 >: x\epsilon X\}$

vii. Neutrosophic empty set: $0_X = \{< x,\ 0,\ 1,\ 1 >: x\epsilon X\}$

Theorem 2.1 [20] Let A and B be two neutrosophic sets on X then

i. A∨A= A and A∧A=A
ii. A∨B= B∨A and A∧B = B ∧ A
iii. A ∨ 0_X= A and A ∨ 1_X=1_X
iv. A∧ 0_X= 0_X and A∧ 1=A
v. A ∨ (B∨C)= (A∨B) ∨C and A ∧ (B∧C)= (A∧B) ∧C
vi. $(A^C)^C$=A

Theorem 2.2 [20] Let A and B be two neutrosophic sets on X then De Morgan's law is valid.

i. $[\vee_{i\in I} A_{i\in I}]^C = \wedge_{i\in I} A_i{}^C$
ii. $[\wedge_{i\in I} A_i]^C = \vee_{i\in I} A_i{}^C$

Definition 2.2 [7] Neutrosophic topological spaces

Let τ be a collection of all neutrosophic subsets on X. Then τ is called a neutrosophic topology in X if the following conditions hold

i. 0_X and 1_X is belong to τ.
ii. Union of any number of neutrosophic sets in τ is again belong to τ.
iii. Intersection of any two neutrosophic set in τ is belong to τ.

Then the pair (X, τ) is called neutrosophic topology on X.

Definition 2.3

Let (X, τ) be a neutrosophic topological space over X and An is neutrosophic subset on X. Then, at that point, the neutrosophic interior of An is the union of all neutrosophic open subsets of A. Obviously neutrosophic interior of An is the biggest neutrosophic open set over X which containing A.

Definition 2.4

Let (X, τ) be a neutrosophic topological space over X and An is neutrosophic subset on X. Then, the neutrosophic closure of An is the intersection of all neutrosophic shut super sets of A. Obviously neutrosophic closure of An is the littlest neutrosophic shut set over X which contains A.

Main Results

Definition 3.1

A system (X, τ_i, τ_j) consisting of a set X with two neutrosophic topologies τ_i and τ_j on X is called Neutrosophic Bitopological space. Throughout in this paper the indices i, j take the value $\in \{1, 2\}$ and $i \neq j$.

Example 3.1

Let X= {a,b} and $A = \{< a, 0.5, 0.5, 0.5 >, < b, 0.4, 0.4, 0.4 >\}$,
$B = \{< a, 0.6, 0.6, 0.6 >, < b, 0.3, 0.3, 0.3 >\}$, $C = \{< a, 0.6, 0.6, 0.6 >, < b, 0.2, 0.2, 0.2 >\}$,
$D = \{< a, 0.7, 0.7, 0.7 >, < b, 0.4, 0.4, 0.4 >\}$. Then $\tau_1 = \{0_X, 1_X, A, B, A \wedge B, A \vee B\}$ and $\tau_2 = \{0_X, 1_X, C, D, C \wedge D, C \vee D\}$ then (X, τ_1, τ_2) is neutrosophic bitopological space

Definition 3.2

Let (X, τ_i, τ_j) be a neutroscopic bitopological space. Then for a set $A = \{< x, \mu_{ij,} \sigma_{ij}, \gamma_{ij} >: x \epsilon X\}$, neutrosophic (τ_i, τ_j)- N-interior of A is the union of all (τ_i, τ_j)-N-open sets of X contained in A and is defined as follows

(τ_i, τ_j)-N-Int(A) = $\{< x, \vee_{\tau_i} \vee_{\tau_j} \mu_{ij}, \wedge_{\tau_i} \wedge_{\tau_j} \sigma_{ij}, \wedge_{\tau_i} \wedge_{\tau_j} \gamma_{ij} >: x \epsilon X\}$

Note : Here $\mu_{ij,}$ represents degrees of membership function, σ_{ij} is the degree of indeterminacy and γ_{ij} is the degree of non-membership function of a neutrosophic set and i is interrelated with neutrosophic topology τ_i and j is interrelated with neutrosophic topologie τ_j when we discussed on (τ_i, τ_j)-N-Int(A).

Example 3.2

Let X={a, b} and $A = \{< a, 0.5, 0.5, 0.5 >, < b, 0.4, 0.4, 0.4 >\}$, $B = \{< a, 0.6, 0.6, 0.6 >, < b, 0.3, 0.3, 0.3 >\}$, $C = \{< a, 0.6, 0.6, 0.6 >, < b, 0.2, 0.2, 0.2 >\}$, $D = \{< a, 0.7, 0.7, 0.7 >, < b, 0.4, 0.4, 0.4 >\}$. Then $\tau_1 = \{0_X, 1_X, A, B, A \wedge B, A \vee B\}$ and $\tau_2 = \{0_X, 1_X, C, D, C \wedge D, C \vee D\}$ then (X, τ_1, τ_2) is neutrosophic bitopological space
Let $Q = \{< a, 0.6, 0.4, 0.4 >, < b, 0.3, 0.3, 0.4 >\}$
τ_2-N-Int (Q) = 0_X and τ_1-N-Int (0_X) =0_X
Hence (τ_1, τ_2)-N-Int(Q)= 0_X

Theorem 3.1

Let (X, τ_i, τ_j) be neutrosophic bitopological space then

i. (τ_i, τ_j)-N-Int(0_X) = 0_X, (τ_i, τ_j)-N-Int(1_X) = 1_X
ii. (τ_i, τ_j)-N-Int(A)≤ A.
iii. A is neutroscopic open set if and only if A=(τ_i, τ_j)-N-Int(A)
iv. (τ_i, τ_j)-N-Int [(τ_i, τ_j)-N-Int(A)] = A
v. A≤B implies (τ_i, τ_j)-N-Int(A)≤(τ_i, τ_j)-N-Int(B)

vi. (τ_i, τ_j)-N-Int(A)∨ (τ_i, τ_j)-N-Int(B) ≤ (τ_i, τ_j)-N-Int(A∨B)

vii.(τ_i, τ_j)-N-Int(A ∧ B) = (τ_i, τ_j)-N-Int(A) ∧ (τ_i, τ_j)-N-Int(B).

Proof of the theorems are straightforward.

Remark 3.1: (τ_i, τ_j)-N-Int(A)≠(τ_j, τ_i)-N-Int(A) when i≠ j. For this we cite an example.

Example 3.3

Let X={a, b} and $A = \{< a, 0.5, 0.6, 0.7 >, < b, 0.4, 0.5, 0.6 >\}$,
$B = \{< a, 0.6, 0.6, 0.7 >, < b, 0.6, 0.4, 0.5 >\}$, $C = \{< a, 0.6, 0.6, 0.7 >, < b, 0.3, 0.2, 0.3 >\}$, $D = \{< a, 0.7, 0.6, 0.7 >, < b, 0.7, 0.2, 0.3 >\}$. Then $\tau_1 = \{0_X,\ 1_X,\ A,\ B, A \wedge B, A \vee B\}$ and $\tau_2 = \{\ 0_X,\ 1_X,\ C,\ D, C \wedge D, C \vee D\}$ then (X, τ_1, τ_2) is neutrosophic bitopological space.

Let $P - \{< a, 0.8, 0.4, 0.5 >, < b, 0.7, 0.1, 0.2 >\}$

Then τ_2-N-Int(P) = D and (τ_1, τ_2)-N-Int(P) = B.

Now τ_1-N-Int(P) = B and (τ_2, τ_1)-N-Int(P) = C.

Hence the result that is (τ_1, τ_2)-N-Int(A)≠((τ_2, τ_1)-N-Int(A).

Conclusion

In this work we have redefined the definition of Bitopological space with the assistance of netrosophic set. Then we have investigated the properties of interior, closure and limit of neutrosophic bitopological spaces. Trust our work will help in additional investigation of neutrosophic generalized shut sets in neutrosophiv bitopological space. This might lead a fresh start for additional exploration on the investigation of generalized shut sets in neutrosophic bitopological space associated with digraph and directed diagrams. This may likewise prompt the new properties of separation axioms on neutrosophic bitopological space.

References

1. L. A. Zadeh , Fuzzy sets, Information and Control, 8 (1965), 338-353
2. K. T Atanassov., Intuitionistic fuzzy sets, Fuzzy sets and systems, 20. (1986), 87-96,
3. F. Smarandache, Neutrosophic set - a generalization of the intuitionistic fuzzy set, International Journal of Pure and Applied Mathematics, 24(3) (2005) 287–297.
4. F. Smarandache, Neutrosophy and neutrosophic logic, first international conference on neutrosophy, neutrosophic logic, set, probability, and statis- tics, University of New Mexico, Gallup, NM 87301, USA(2002).
5. C. L. Chang , Fuzzy Topological Space, Journal of Mathematical Analysis and Application 24 (1968), 182- 190
6. D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 88 (1997), 81- 89.

7. F. G. Lupi´a˜nez, On neutrosophic topology, The International Journal of Systems and Cybernetics, 37(6) (2008), 797–800.
8. F. G. Lupi´a˜nez, Interval neutrosophic sets and topology, The International Journal of Systems and Cybernetics, 38(3/4) (2009), 621–624.
9. F. G. Lupi´a˜nez, On various neutrosophic topologies, The International Journal of Systems and Cybernetics, 38(6) (2009), 1009–1013.
10. F. G. Lupi´a˜nez, On neutrosophic paraconsistent topology, The International Journal of Systems and Cybernetics, 39(4) (2010), 598–601.
11. A. Salama and S. AL-Blowi, Generalized neutrosophic set and generalized neutrosophic topological spaces, Computer Science and Engineering, 2(7) (2012), 129–132.
12. J. C. Kelly, Bitopoligicaal spaces, Pro. London math soc., 313 (1963), 71-89
13. A. Kandil, Nouth A. A., El-Sheikh S. A., On fuzzy bitopological spaces, Fuzzy sets and system, 74(1995) 353-363
14. J. Lee, J. T. Kim, Some properties of Intuitionistic Fuzzy Bitopological Spaces, SCIS-ISIS 2012, Kobe, Japan, Nov. 20-24.
15. P. M. Pu and Y. M. Liu, Fuzzy topology I: neighbourhood structure of a fuzzy point and Moore-Smith convergence, J. Math. Anal. Appl. 76, (1980) 571–599
16. R. H. Warren, Boundary of a fuzzy set, Indiana Univ. Math. J. 26(1977), 191–197 17. M.
17. Abdel-Baset, R.. Mohamed, A. E. N. H. Zaied & F. Samarandache, A Hybride plitogenic decisionmaking approach with quality function deployment for selecting supply chain sustainability metrics, Symmetry, 11(7)(2019), 903.
18. M. Abdel-Baset, G. Manogaran, A. Gamal & F. Samarandache, A Group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection , Journal of Medical System, 43(2)(2019), 38.
19. M. Abdel-Baset & R. Mohamed, A Novel plithogenic TOPSIS-CRITIC model for sustainable supply chain risk management, Journal of Cleaner Production, 247(2020), 119586.
20. Wang H, Smarandache F, Zhang YQ, Sunderraman R Single valued neutrosophic sets, Multispace and Multistructure, 4(2010): 410-413.